Investigation results oriented to solving the problem of automating gear and gear cutting tool design are discussed. Program for graphic – analytical and analytical computation of gear cutting tool with modified tooth profile as well as computer program for gear forming mold used, die for manufacturing injection molded plastic gears, are considered. Programming and computing suite for computation of gear complete with multipair meshing for tractor “Belarus” gearbox is presented.

**Keywords:** CAD-systems, programming and computing suite, gear cutting tool with modified profile, gear forming mold die, multipair meshing.

**Introduction**

The article presents the results of development of software products which allow to automate the gear drive and gear forming tool calculation for manufacturing gears both by mechanical processing and injection molding. The studies have been carried out by the Joint Institute of Mechanical Engineering of the National Academy of Sciences of Belarus, Metal-Polymer Research Institute of the National Academy of Sciences of Belarus and Minsk Tractor Plant. The developed computer programs provide for optimization of tooth meshing by criterion of contact ratio maximization; designing the gear cutting tool with modified tooth profile; designing plastic cylindrical and bevel gear drives and computation of forming tools (mold dies) for manufacturing plastic gears. All of this produces higher quality results and cost reduction at the gear design, manufacturing and operation stages.

Let’s analyze the modern state of software products development. It should be noted that currently the market of CAD/CAM/CAE systems offers a wide variety of software products of various levels and for various purposes. Well known software products, such as Pro/Engineer, Unigraphics, CATIA, EUKLID, I-DEAS, Ansys, T-FLEXCAD, APM WinMachine, Solid Works, Compas, AutoCAD, have special modules which provide for computing and building models of various types of gear wheels and drives, including the ones with plastic gears.

APM WinMachine system (offered by Research and Software Development Center APM Ltd., Russia) is equipped with APM Trans module which allows to perform design and verification calculations of the gear drives by meshing (external and internal meshing cylindrical gear drives, external meshing helical gears, herringbone, bevel gears with straight and circular arc tooth, worm gears). It is reported [1] that a new APM WinMachine 8.5 release offers the following features: algorithms of calculation procedures, the procedure of geometrical calculation of non-equally shifted bevel gears is included, tolerance assignment system is developed, emphasis is placed on developing
English version of the program. It is planned to take measures on supporting AGMA, ISO, DIN and JIS calculation standards applied in the countries, which determine the world's engineering policy, as well as steps on APM WinMachine system integration with such computer graphics products as Solid Edge, AutoCAD, Inventor and ADEM. The company intends to complement APM Tran’s module with plastic gear drives computation; according to the authors' data, a tool for calculation of such drives is currently not available at CIS software market.

Special software products designed for gear drives calculation are based primarily on standard calculations according to the national (DIN 3990, AGMA 610) and international (ISO 6363) standards and may have additional features specific to their use. ZABGON program [2] provides possibility to design gear drives with non-standard parameters. The authors [3] offer a suite of programs to support a component's life cycle - from the design to the manufacturing stage. The program [4] performs AutoCAD-aided calculations taking into account dynamic loads; besides, the gear drive durability is calculated. Programs [5, 6] supplementing AutoCAD, Compas, etc. are offered to calculate geometry, strength and reliability of external and internal meshing cylindrical spur and helical gear drives, bevel gears with spur and circular arc teeth, cylindrical worm gears.

Software product GEAR Trax developed by Camnetics Inc., USA, is designed to create models of transmission system parts – external and internal meshing cylindrical spur and helical gears, bevel gears with spur and circular arc teeth, elements of Archimedean worm gearing. GEAR Trax interacts with Solid Works, Invектор, Solid Edge software.

Non-standard approach to creation of CAD software for gear drives is proposed by the authors [7, 8]. The concept is based on the use of software modules which do not depend upon the gear drive type and which support the process of drive designing, manufacturing and operation. The library of universal software modules should include the following modules: 3D modeling and interactive visualization; gear drive scheme module; gear cutting process; tooth contact modeling; loaded tooth contact analysis; simulation of pinion and gear running-in which computational models are based on the results of measurements of real teeth surfaces; predicting the gear drive condition (wear, noise, vibration activity).

A variant of automatic designing the externally meshing cylindrical gear drives based on the concept of dynamic blocking circuit is presented in work [9]. This option enables the user to select rational coefficients for the pinion and gear shifts and carry out a forecast-based designing of a gear drive by assuring, at the same time, high efficiency and visibility of the design process.

A number of software products for gear drives designing have been developed as a part of academic activities (term projects). The authors [10] developed a program for designing one- and two-stage reducers for the expanded scheme, scheme with low-speed stage bifurcation, coaxial scheme. All types of calculations for gear drives and shafts are done, bearings selection is performed; as the result, the reducer structural design is prepared. TransMECH software developed by Machine Parts and Lifing-and-Shifting Machines and Mechanisms department (BNTU) is based on the materials of the reference guide [11]; it assures the drives designing in compliance with GOST 21354 standard with minimal computer resources.

In software products which have sections directly related to designing the plastic gears (in particular, such well-known programs as KISS SOFT, StarGear [12], “Plastic Gearing” Software program of ABA-PGT, Inc. Company, etc.) the design algorithm is based, primarily, on ISO 6336, DIN 3990 and AGMA 610 standards. The concept of taking into consideration specific features of plastics during the design is described to the fullest extent possible, in particular, in VDI 2545 specifications [13], however, availability of the sufficient for calculations data on mechanical properties of thermoplastics and design factors which assure end-to-end computer-aided design is limited to the information on polyamides PA6, PA66, glass-filled PA6 and polyoxymethylene; for the rest of plastics such data is either not available or for some of them it is proposed to use simplified calculations based on the load factor $c = F/b_0 \cdot m$.

A set of basic racks (4 modifications with various tooth heights) proposed by ABA-PGT Inc. assures equal-sized thickness at the tooth addendum of a pair of mating gears and modifi-
cation of the profile at the top of tooth, excludes the teeth undercutting and decreases bending stresses owing to completely rounded tooth space; the calculation program is also available [14]. The basic rack parameters are standardized [15].

Methods and program for calculation of load carrying capacity of the gear drive by safety criteria along with determination of safety factors for the tooth fatigue fracture, contact endurance, wear, temperature exceeding the temperature of plastic gear material melting under long-term continuous operation, as well as for lubricating material temperature are described (by the example of metal-polymer worm drive with plastic helical gear) in works [16, 17].

The authors [18, 19] present a variant of gear drives calculation (including the ones with plastic gears) based on generalized parameters under the brand name of Direct Gear Design supplemented (for plastic gears) with the Genetic Mold Solution concept from which it follows that the form of the plastic gear being made by injection molding of thermoplastic polymer or composite material on its base (as well as any other product) contains, figuratively speaking, "genetic information" on the history of production of materials forming the mold die surface and parameters of technological molding process on automatic molding machine of a certain type. The AKGears-developed software product provides for calculation of various types of gear drives with optimal shape of teeth, optimization of the form of fillet by the criterion of minimizing bending stresses, as well as drives with asymmetric teeth profiles assuring the increased load carrying capacity of the drive.

Optimization of meshing parameters of a gear pair of tractor “Belarus” gearbox using IOSO technology

The problem of optimization of a gear pair meshing parameters has been set in view of the necessity to create a tractor “Belarus” gearbox of the increased load carrying capacity while retaining its weight-and-dimensional characteristics (namely, center distances of gear pairs) and speed range (i.e. gear ratios).

Retention of the body parts geometrical parameters is caused by the necessity to assure block-modular principle of the design which provides for creation of various modifications of tractors assuring their technological versatility in the expanded traction range.

Such problem statement implies that a solution can be found by development of a modified meshing with a basic rack which assures multipair meshing, i.e. participation of two-three simultaneously meshed teeth pairs.

To solve the problem an effective IOSO technology was used which is based on patented algorithms, can be integrated with various CAE/CAD applications (which provides for its usage at the engineer’s desktop without a need for additional programs) and does not require from the user to be an expert mathematician.

1. Concept of the optimization project

The optimization problem appears as follows: it is required to maximize the transverse contact ratio, limit the specific sliding in lower point of pinion and gear profile while assuring retention of center distances between gear pairs, gear ratio, strength condition, durability and condition for a gear pair existence. The following optimization parameters are taken: coefficients of displacement, number of teeth and addendum coefficient.

Irrespective of the optimization problem, when IOSO technology is used it shall be necessary to have a mathematical model of an optimization object which describes operation and behavior of such object.

When creating the optimization project, the mathematical model of the optimization object is integrated with IOSO software suite (the process is enabled by IOSO suite built-in tools). After integration IOSO suite interacts with the mathematical model by using two types of text files – input parameters file and output parameters file.

The following iteration process occurs when searching for optimal solutions of the mathematical model: IOSO suite generates (in automatic mode) the values of input parameters and writes them into the input data file, then IOSO package runs the mathematical model which writes the calculated output parameters (criteria) into the output data file; IOSO suite reads the output data file and analyzes the obtained results, then IOSO algorithm takes a decision on changing the input parameters.
2. **Gear pair mathematical model**

At creating the mathematical model of a gear pair of a gearbox the following calculation programs have been used:

1. Program for geometrical calculation of involute gears according to GOST 16532-70 - Irion ZGEOM (see Fig. 1). The program input (initial) parameters are: number of teeth on the gear wheel, module, coefficients of displacement, teeth angle, parameters of counterpart rack profile and gear wheel face width. Geometrical parameters of the meshing are output results of calculation performed by Irion ZGEOM. Besides, the program runs a check on absence of interference, undercutting and tooth tapering of gear wheels. The program can read input parameters from a text file and write output parameters into a text file.

![Fig. 1. Irion ZGEOM main window](image1)

2. Program for strength calculation of externally meshing involute cylindrical gear drives according to GOST 21354-87 – Irion PZub (see Fig. 2). The program input (initial) parameters are: results of geometry calculations performed by Irion ZGEOM, gear drive load mode, parameters of material, basic parameters of gearing and additional gear ratios. Safety factors for contact and bending durability are output results of calculation performed by Irion PZub. The program can read input parameters from a text file and write output parameters into a text file. Irion PZub takes into account multi-stage gear drive loading mode according to the methodology of equivalent stresses described in Appendix 3 to GOST 21354-87.

![Fig. 2. Irion PZub main window](image2)

3. Program for calculation of coefficients of displacement and number of teeth on the gear wheel under the specified center distance and gear ratio - Irion Get_Xn. The program is implemented according to the methodology described in GOST 16532-70. Irion Get_Xn has no user interface, it reads input parameters from a text file and writes output parameters into a text file.

Thus, the mathematical model includes calculation of geometrical parameters of the gear wheel ($x_2$ and $z_2$ when $x_1$, $z_1$, $a_w$ are specified) by Irion Get_Xn, gear pair geometry calculation by Irion_ZGEOM and strength calculation by Irion_PZub.

As a matter of experience in designing gear drives, a design engineer shall specify maximum deviations from the required gear ratio and a pair of gear wheels and the variation range for addendum coefficient $h_a$.

3. **Integration with IOSO technology**

The created mathematical model of the gear pair is integrated with the optimization technology software suite. Input optimization parameters, parameters to be transmitted, input and output files, as well as IrionGet_Xn, Irion_ZGEOM, Irion_PZub programs for calculation of gear meshings are specified using the IOSO suite built-in tools.

Input and output parameters are adjusted and their existence range is set. Independent input parameters are: $x_1$, $z_1$, $m$, $h^*$. Dependent input parameters are: $\beta$, $\alpha$, $h_a$, $c$. The following parameters are taken as the output ones: $x_2$, $z_2$, $\varepsilon_0$ (parameter which shall be maximized); specific sliding on the pinion ($V_{p1}$) and gear wheel ($V_{p2}$); center distance ($a_w$); limitations on undercutting...
of the pinion \((pod_1)\) and gear wheel \((pod_2)\), tapering of teeth on the pinion \((Zos_1)\) and gear wheel \((Zos_2)\), interference of two types on the pinion \((int_{11}, int_{12})\) and gear wheel \((int_{21}, int_{22})\); safety factors at calculation of contact \((H_1, H_2)\) and bending strength \((F_1, F_2)\); safety factors at calculation by maximum contact \((H_{max1}, H_{max2})\) and maximum bending stresses \((F_{max1}, F_{max2})\).

4. Example of the calculation implementation in the program

The initial data for optimization are shown in Tables 1 and 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Label</th>
<th>Numerical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pinion number of teeth</td>
<td>(z_1)</td>
<td>19</td>
</tr>
<tr>
<td>Gear wheel number of teeth</td>
<td>(z_2)</td>
<td>45</td>
</tr>
<tr>
<td>Module, mm</td>
<td>(m)</td>
<td>4.5</td>
</tr>
<tr>
<td>Pinion addendum modification coefficient</td>
<td>(x_1)</td>
<td>0.444</td>
</tr>
<tr>
<td>Gear wheel addendum modification coefficient</td>
<td>(x_2)</td>
<td>0.084</td>
</tr>
<tr>
<td>Helix angle</td>
<td>(\beta)</td>
<td>0</td>
</tr>
<tr>
<td>Pressure angle</td>
<td>(\alpha)</td>
<td>20</td>
</tr>
<tr>
<td>Addendum coefficient</td>
<td>(h^*a)</td>
<td>1</td>
</tr>
<tr>
<td>Coefficient of boundary height</td>
<td>(h^*l)</td>
<td>2</td>
</tr>
<tr>
<td>Bottom clearance coefficient</td>
<td>(c^*)</td>
<td>0.25</td>
</tr>
<tr>
<td>Pinion facewidth, mm</td>
<td>(b_1)</td>
<td>27</td>
</tr>
<tr>
<td>Gear wheel facewidth, mm</td>
<td>(b_2)</td>
<td>25</td>
</tr>
<tr>
<td>Centre distance, mm</td>
<td>(a_w)</td>
<td>146.25</td>
</tr>
<tr>
<td>Gear ratio</td>
<td>(u)</td>
<td>45/19</td>
</tr>
<tr>
<td>Contact ratio</td>
<td>(\varepsilon_c)</td>
<td>1.4</td>
</tr>
<tr>
<td>Yield stress, MPa</td>
<td>(\sigma_y)</td>
<td>736</td>
</tr>
<tr>
<td>Contact stress, MPa</td>
<td>(\sigma_{Hlim})</td>
<td>1426</td>
</tr>
<tr>
<td>Bending stress, MPa</td>
<td>(\sigma_{Flim})</td>
<td>950</td>
</tr>
<tr>
<td>Surface roughness parameter</td>
<td>(R_a)</td>
<td>2.5</td>
</tr>
<tr>
<td>Gear accuracy</td>
<td>Accuracy</td>
<td>7-6-6-Cc</td>
</tr>
</tbody>
</table>

Table 2. Load mode

<table>
<thead>
<tr>
<th>Drive No.</th>
<th>Rotational torque, N(\cdot)m</th>
<th>Rate of rotation, min(^{-1})</th>
<th>Service life, h</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>208</td>
<td>2100</td>
<td>6</td>
</tr>
<tr>
<td>13</td>
<td>312.6</td>
<td>2100</td>
<td>24</td>
</tr>
<tr>
<td>19</td>
<td>527.5</td>
<td>2100</td>
<td>241</td>
</tr>
</tbody>
</table>

It is required to select a pair of gear wheels with maximum contact ratio under the specified gear ratio \(u = 2.368421\) with deviation not exceeding 2.5%.

The addendum coefficient range is \(h_a = 1.1\text{–}1.25\). Calculation is performed for four variations of gear meshing’s with various combinations of \(z, z_2, m, h_a\).

As the result of more than 700 iteration a solution has been found with the following value of contact ratio: \(\varepsilon_c = 2.1498\) (which is 53.5% higher than the initial value before optimization \(\varepsilon_c = 1.4\)). History of the search for solution is presented in Fig. 3 and the results of optimization are shown in Fig. 4.

Fig. 3. Contact ratio vs. number of iterations

![Fig. 3. Contact ratio vs. number of iterations](image)

Fig. 4. Optimization results

![Fig. 4. Optimization results](image)

PC-aided design of substandard gear cutting tool

The majority of gear wheels of modern gearboxes are made with special modification of a tooth profile, namely, with smooth deviation on the tooth addendum and addendum from the theoretical involute of the main profile “towards the minus” which leads to the decrease of noise level and minimization of the friction work and wear rate.

“Graphicoanalytical method for calculation of cutting tools for cylindrical gears with involute modified tooth profile” program (hereinafter referred to as GRIN) provides solution of both the direct problem (tool profile calculation based on the given gear tooth profile) and the
reverse problem (verification of the gear tooth profile based on the available tool profile). The program enables calculation of all types of tools for tooth cutting by rolling: gear hobs, gear-shaping cutters, shavers, and worm grinding wheels. To assure GRIN software operation it is required to have AutoCAD2000 installed.

GRIN software is used for PC-aided designing of a modified profile of cutting tools which process the gear teeth by rolling, as well as for designing a profile of gear spline hobs for cutting straight-sided splines. The program was created in AutoLisp programming language in the AutoCAD environment. It provides possibility to track the rolling process on the monitor, determine the elements of the teeth profile parameters which practically do not lend themselves to analytical calculation.

“Analytical calculation of cutting tools for cylindrical gears with involute tooth profile” program (hereinafter referred to as ARIN) is used for PC-aided designing of modified profile of cutting tools which process the gear teeth by rolling. The program was created in FORTRAN programming language. The software enables the user to perform multi-variant analysis of the teeth-cutting results. Assessment of feasibility of achieving the required result taking into account conditions and possibilities of production can be performed already at the stage of cutting tools design; the program allows to accurately and promptly track the influence of changes in the tool profile dimensions and maximum dimensions of the part being processed upon parameters of the modified tooth profile of the product.

Besides, GRIN and ARIN programs include calculation of gear-shaping cutters for externally and internally meshing involute gear wheels, gear hobs for processing spline shafts. GRIN and ARIN programs allow to quickly and with a high degree of accuracy determine optimal parameters of modified profiles. These programs provide for high-precision determination of parameters of the transition curve at the tooth dedendum and prevention of teeth undercutting which has a significant impact upon the gear drive durability.

PC-aided design of drives with plastic gears

1. *Purpose and subject of the program*

The program modules are designed for PC-aided designing of involute cylindrical and bevel drives with plastic gear wheels, basic rack as per GOST 13755-81 at \( m > 1 \) and GOST 9587-81 at \( m < 1 \), for cylindrical gear drives as per GOST 13754-81 at \( m \geq 1 \)mm. Methodology of calculation of nominal dimensions of gear wheels and drives with constant radial clearance at the crossed axes angle of 90° is implemented for bevel gear drives.

The geometrical parameters limit values are:
- Module, \( m \sim 0.1 \) to 10 mm;
- (Mean) reference diameter, \( d \) – up to 400 mm;
- Center distances, \( a_w \sim 250 \) mm at \( m > 1 \) and up to 180 mm at \( m < 1 \);
- Mean cone distance – up to 200 mm.

In the process of designing the program runs the design calculation of the gear drive and calculation of nominal dimensions and geometrical parameters of gear wheels; calculation of the respective parameters is performed according to the specified grade of accuracy; quality of engagement is evaluated, animated visual control over the gear wheels meshing process is implemented; full calculation protocol is generated; data for drawings are outputted; verification strength calculation based on contact strength and bending fatigue strength and calculation of gear forming mold dies (based on various variants of initial data) is performed which results are used as the data for their manufacturing at CNC machines. When plastic gear wheels are calculated according to the program, the following factors which take into account the specific features of plastic materials at various stages of the gear drives designing, manufacturing and subsequent operation are considered:
- taking account of the products shrinkage at calculation and manufacturing of the gear forming mold dies according to various variants of the initial data;
- calculation of real measuring dimensions and quality indicators taking into account the specified accuracy parameters;
- taking account of tooth surface temperature and bulk temperature at the tooth root in strength calculations;
- significant dependence of mechanical properties on temperature and time of operation.
(the database is available for the elasticity modulus and endurance limits at calculation of contact endurance and teeth bending fatigue strength for certain structural polymer materials which can be replenished as more experimental data are received);

- possibility of calculating the geometry of gear wheels with substandard parameters of the basic rack.

Nominal dimensions of cylindrical gear drive and gear wheels are calculated as per GOST 16532-83, bevel gear drive and bevel gear wheels – as per GOST 19624-74.

Basic rack parameters are the initial data; center distance \( a_0 \) or coefficients of the pinion \( x_1 \) and gear \( x_2 \) displacement are factors for calculation of cylindrical gear drive, while coefficients of tangential displacement \( x_{1t} \) and \( x_{2t} \) are additional factors for calculation of a bevel gear drive.

At a given \( a_0 \), it is possible to apply GOST 16532-83 recommendations for breaking the coefficient of the sum of displacements \( x \) sum into \( x_1 \) and \( x_2 \) constituents.

The tolerances are selected automatically in accordance with the accepted grades of accuracy as per norms of kinematic accuracy, smoothness of operation, teeth contact, type of gear mating and type of backlash tolerance as per GOST 1643-81 at \( m > 1 \) and GOST 9178-81 at \( m < 1 \) for cylindrical gear drives and GOST 1758-81 at \( m > 1 \) mm and GOST 9368-81 at \( m < 1 \) mm for bevel gears drives. Conditions assuring correct measurements and the gear drive quality parameters taking into account the tolerances are verified.

Procedure of strength calculations (including the design and verification calculations) meets GOST 21354-85 requirements. Strength calculation uses the basic calculation dependencies to determine contact strength of active tooth flanks and teeth bending strength as specified in this standard. Methodological guidelines [20] are used for calculation of bevel gear drives strength.

Calculation of gear forming mold dies and punching electrodes for cylindrical and spur bevel plastic gear wheels is performed. Calculation is made using the initial parameters indicated in the gear wheel drawing or the program data exchange file and the information on mold shrinkage presented as an array of measurement data on the preproduction series of imitators of a gear wheel being designed or as an interval (maximum deviations) of the shrinkage.

The program calculates:
- mean value and confidence interval of the molding shrinkage at a given reliability coefficient; besides, if the dispersion of shrinkage values does not provide for obtaining a gear wheel of the specified accuracy, the user gets a message on acceptable shrinkage interval;
- mold die geometrical parameters, its controllable dimensions and tolerances;
- coordinates of upper, middle and lower tooth profiles of a mold die and gear wheel which involute areas are approximated with circular arcs of specified accuracy;
- coordinates of teeth profile of a punching electrode for manufacturing mold dies of helical and bevel gear wheels with the operator-defined displacement (interelectrode gap) with respect to a mean profile of mold die teeth;
- file with codes for generating a program for manufacturing spur mold die with a wire electrode on EDM machine.

The calculated coordinates of profiles are written into *.DXF data exchange file which allows to display a mold die drawing in the graphics editing system (e.g. AutoCAD). By default, the program envisages that the basic (one of the most common) variant of the initial data is specified in the initial data related to basic rack parameters and indicators of gear wheels accuracy; besides, when selecting the basic rack parameters the operator receives prompts with possible standard values of the basic rack parameters and when assigning grades of accuracy and types of gear mating - prompts on limitations on combination of norms of kinematic accuracy, smoothness of operation and norms for teeth contact of gear wheels and gear drives of various degrees of accuracy envisaged by the standards.

Software program “PC-aided design of cylindrical gear drives with plastic gears” is registered in the RB Center of Intellectual Properties under No. 370 [21].

2. **Updating accuracy parameters of molded plastic gears**

The correspondence of dimensional deviations of molded plastic articles to required accuracy indices is known to be ensured by correct accounting of mold shrinkage and its derivations when calculating mold die (or mold
punch) dimensions. For the molded plastic gears, the dimensions of the forming mold die could be assigned in accordance with the mold shrinkage interval ($S_{min...S_{max}}$) indicated in reference literature, or by mean shrinkage $\bar{s}$ and its deviations $\pm \Delta s$ calculated by the measurement results of gear imitators, or by shrinkage components, which are calculated by the direct measurement results of pilot batch preproduction series of gears.

A methodology of updating gear die dimensions is considered in the article based on the data received at measuring preproduction series of gears of electrogrinder drive and concrete examples are analyzed. Provision for compliance of the real dimensions to required accuracy indices has been examined.

3. **Procedure of mold shrinkage computation**

Computation is based on determining mold shrinkage components in terms of the base geometrical parameters, which define gear dimensions in the system of generalized parameters [22]. Within this system, the base geometrical parameters of the tooth ring are computed independently of the parameters of the basic rack tooth profile.

As independent parameters, the base diameters $d_a^s,d_a^m$, angular thicknesses on the base circle $\theta_a^s,\theta_a^m$ and tip diameters $d_b^s,d_b^m$ of the gear (index “a”) and mold die (index “m”) are accepted. In correspondence to the generalized parameters, the coefficients of mold shrinkage $S_a, S_b, S_c$ are computed [23]:

$$S_a = \frac{(d_a^m - d_a^s)}{d_a^m}; \quad (1)$$

$$S_b = \frac{(d_b^m - d_b^s)}{d_b^m}; \quad (2)$$

$$S_c = \frac{(\theta_a^m - \theta_a^s)}{\theta_a^m}. \quad (3)$$

Parameter $d_a^s$ is directly measured. To compute the remaining ones $d_a^m$ and $\theta_a^m$, one of the following methods described earlier in [23-25] is used:

- Mold shrinkage computation by coordinates of tooth profile points [24];
- Computation of involute parameters by measuring dimensions over balls at different diameters [23];
- Method of difference of the base tangent lengths [24, 25].

4. **Computation procedure of probability of conformance between real accuracy indices and sought ones**

Deviation of any actual mean controlled dimension $\bar{X}$ from a specified mean dimension in drawing $\bar{X}^*$ is compared to confidence interval $\Delta X$ for the mean one

$$|\bar{X} - \bar{X}^*| < \Delta X / 2 \sqrt{n},$$

on the base of comparing the deviation of dimension $\bar{X}$ from $\bar{X}^*$ with tolerance $T_b$, the probability of obtaining gears of required accuracy is evaluated

$$P(X) = \Phi_s(z_i) - \Phi_s(z_e) \geq P(X)^i$$

where

$$Z_i = \frac{\bar{X}^* + T_b/2 - \bar{X}}{\sigma_s}; \quad Z_e = \frac{\bar{X}^* - T_b/2 - \bar{X}}{\sigma_s}$$

where $\sigma_s$ – mean square deviation confidence.

Confidence interval is computed by equation:

$$\Delta X = \bar{X} \cdot t(\alpha),$$

where $t(\alpha)$ – Student’s factor.

All calculations are provided at probability 0.90.

5. **Computation of updating corrections**

To compute the forming die geometrical parameters, update the results of measuring preproduction series of gears, the following dependencies are used [23]:

Mean tip diameter $\bar{d}_a^s$:

$$\bar{d}_a^s = \bar{d}_b^s / (1 - \bar{S}_a).$$

(7)

Mean base diameter $\bar{d}_b^m$:

$$\bar{d}_b^m = \frac{-m}{ \bar{d}_b^m} / (1 - \bar{S}_b).$$

(8)

Module $m^m$:

$$m^m = \frac{-m}{ \bar{d}_b^m} / z \cos \alpha.$$  

(9)

Base diameter $d_b$ will be equal to

$$d_b = \frac{z}{W} \left( \frac{w_{n+1} - w_n}{z_{n+1} - z_n} \right).$$

(10)

Angular tooth thickness $\theta_b$ on the base circle equals to

$$\theta_b = 2 \left[ \frac{W}{d_b} \frac{\pi (z - 1)}{z} \right].$$

(11)

6. **Experimental results and their discussion**
Spur gears made of polyamide 6, trade mark PA6-LT4-CV30-P filled by 30% glass fiber are used as an experimental example (see Fig. 5). The geometrical parameters and accuracy level of experimental gears are presented in Table 3.

The computer software for computation and updating mold die parameters is a constituent part of a software package [26] for calculation of engineering involute gear drives with plastic gears. The software conception is considered on a concrete example in paper [27, 28].

The base diameter of mold die \( d_b \) and angular tooth thickness \( \theta_b \) on the base diameter are computed at availability of two chords \( y_{S_i} \) and \( y_{S_j} \), arranged at corresponding distances \( h_i \) and \( h_j \) from the tooth tip in accordance with the procedure [29].

![Fig. 5. Experimental samples of plastic gears](https://example.com/fig5.png)

**Table 3. Geometrical parameters of preproduction series of gears**

<table>
<thead>
<tr>
<th>№</th>
<th>Parameter</th>
<th>Notations</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Normal module</td>
<td>( m_n )</td>
<td>1.5</td>
<td>1.5</td>
<td>2.0</td>
<td>2.0</td>
<td>2.5</td>
<td>2.5</td>
</tr>
<tr>
<td>2</td>
<td>Tooth number</td>
<td>( z )</td>
<td>12</td>
<td>36</td>
<td>15</td>
<td>32</td>
<td>15</td>
<td>33</td>
</tr>
<tr>
<td>3</td>
<td>Addendum modification coefficient</td>
<td>( x )</td>
<td>+0.3</td>
<td>-0.3</td>
<td>+0.3</td>
<td>-0.3</td>
<td>+0.3</td>
<td>-0.3</td>
</tr>
<tr>
<td>4</td>
<td>Nominal tip diameter</td>
<td>( d_a )</td>
<td>21.9</td>
<td>56.1</td>
<td>35.2</td>
<td>66.8</td>
<td>44.0</td>
<td>86.0</td>
</tr>
<tr>
<td>5</td>
<td>Mean tip diameter in the drawing</td>
<td>( \bar{d}_a )</td>
<td>21.795</td>
<td>55.95</td>
<td>35.12</td>
<td>66.65</td>
<td>43.92</td>
<td>85.825</td>
</tr>
<tr>
<td>6</td>
<td>Nominal base tangent length over ( z_n ) teeth</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
                 \( \bar{w}_n^g \) | 7.20| 15.497| 9.687| 15.247| 12.109| 19.094|
| 7 | Mean base tangent length over \( z_n \) teeth in the drawing |
                 \( \bar{g}_n^* \) | 7.0023| 15.7473| 9.4870| 15.0466| 11.9087| 18.8633|
| 8 | Number of enclosed teeth for \( \bar{w}_n^g \) |
                 \( z_n \) | 2 | 4 | 2 | 3 | 2 | 3 |
| 9 | Mean base tangent length over \( z_{n+1} \) teeth in the drawing |
                 \( \bar{g}_{n+1} \) | 11.4303| 20.1754| 15.3912| 20.9508| 19.2890| 26.244|
| 10 | Number of enclosed teeth for \( \bar{w}_{n+1}^g \) |
                 \( z_{n+1} \) | 3 | 5 | 3 | 4 | 3 | 4 |
| 11 | Base diameter                      | \( d_b^g \) | 16.9145| 50.7434| 28.1908| 60.1403| 35.2385| 77.5246|
| 12 | Pitch                              | \( p_a \)   | 4.4282| 4.4282| 5.9042| 5.9043| 7.3803| 7.3803|
| 13 | Grade of accuracy in accordance with Standard GOST 1643-81 |          | 10-B |

Results of experimental data processing at measuring preproduction series of plastic gears are presented in the view of measuring data. On the base of measurement results the mean controlled dimensions \( \bar{X} (\bar{d}_a, \bar{w}_n, \bar{w}_{n+1}, \bar{p}_a) \), mean
squire deviations \( \sigma_x, \sigma_{w1}, \sigma_{w2(1)}, \sigma_{p1} \), and confident interval \((\Delta d_{m}, \Delta w_{n}, \Delta w_{n+1}, \Delta p_{m})\) are calculated. Updating of base mold die parameters \(d_w, d_0, M_D^0\) and other is computed by equations (7)-(11).

Within the frames of computation we have received updated dimensions of mold dies, which probability increases from 0…0.5 to 0.335…0.999 (see Fig. 6). Low probability of obtaining some part of suitable gears is conditioned by high scatter of mold shrinkage, which does not fit the required tolerance zone. This factor could be perfected by ensuring stable parameters of the technological process of the products.

**Conclusion**

Possibility to design gear drives optimized by criterion of transverse overlap maximization using IOSO technology is presented.

Possibilities of the design using the developed computer programs, gear cutting tools (gear hobs, gear-shaping cutters, shavers, worm grinding wheels) for gear wheels with modified tooth profile providing for calculation of tools parameters promptly and with high degree of accuracy are shown.

The PC-aided design algorithm is proposed and schematic design of the calculation program according to the existing standards taking into account dependencies of thermophysical, physico-mechanical and triboengineering characteristics of plastics and design factors on the type of material of a pair of mating gears, external factors and operation conditions is created.

In addition to computer program for designing gear drives with plastic gears and gear forming mold dies for their manufacturing, a method for PC-aided computation of updating corrections for tooth profile and gear wheel dimensions assuring considerable increase in accuracy factors is proposed. The correction is performed based on measurement of preproduction series of gears.

**Fig. 6, a. Probability of manufacturing gears of required accuracy for tip diameter \(d_0\) before and after updating**

**Fig. 6, b. Probability of manufacturing gears of required accuracy for dimension over pins \(M_D\) before and after updating**

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АВТОМАТИЗИРОВАННОЕ ПРОЕКТИРОВАНИЕ ЗУБЧАТЫХ ПЕРЕДАЧ И ФОРМООБРАЗУЮЩЕГО ИНСТРУМЕНТА

В.Е. Старжинский, В.В. Супин, В.Е. Антонюк

Старжинский Виктор Евгеньевич – д.т.н., главный научный сотрудник отдела «Технология полимерных композитов», Институт механики металло-полимерных систем им. В.А. Белох Нац. Научной академии наук Беларуси, ул. Кирова, 32-А, 246050, г. Гомель, Беларусь

Супин Владимир Викторович – к.т.н., начальник КБ инженерной графики и расчетов УКЭР-1 ПО «Минский тракторный завод», ул. Долгобродская, 29, 220070, г. Минск, Беларусь

Антонюк Владимир Евгеньевич – д.т.н., главный научный сотрудник, Объединенный Институт машиностроения НАН Беларуси, ул. Академическая, 12, 220072, г. Минск, Беларусь

Обсуждаются результаты исследований, направленных на решение проблемы автоматизации проектирования зубчатых передач и формообразующего инструмента для изготовления зубчатых колес. Приведено описание основных положений компьютерных программ для графо-аналитического и аналитического расчета зуборезного инструмента с модифицированным профилем зуба, а также программы для проектирования формообразующих матриц для изготовления методом лития под давлением пластмассовых зубчатых колес. Представлен также программно-расчетный комплекс для расчета комплекта зубчатых пар с многопарным зацеплением коробки передач трактора "Беларусь".

Ключевые слова: CAD-системы, программно-расчетный комплекс, зуборезный инструмент с модифицированным профилем зуба, формообразующая матрица, многопарное зацепление.

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