УДК 621.01

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SOME ISSUES ON INERTIA PROPULSION MECHANISMS USING TWO CONTRA-ROTATING MASSES

1. INTRODUCTION

Among several physical principles that are potentially applicable to produce spatial motion, inertial forces possess an important position. This paper performs the mathematical analysis of the earliest patents that use contra-rotating masses moving along two circumferences on a vertical plane. In addition, the analysis is followed by many comments and lemmas that may be useful in the future research on this technical field.

A report published by NASA in December 2006, which refers to research within the years 1996-2002, has concluded that although it is possible to achieve the temporal lift of an object through several mechanical means, the total impulse becomes zero and, finally, the gravity forces call the object back to the earth [1]. In the western literature, the first relevant patent is probably due to Norman L. Dean [2], which has been called '*Dean's drive*' (http://en.wikipedia.org/wiki/Dean_drive). However, a careful survey reveals that Russian scientists had earlier started and systematically conducted relevant research [3-7], which remains still alive by others [8].

The concept is to utilize contra-rotating eccentric masses for achieving propulsion. However, when the masses rotate along the circumference of a circle on a vertical plane, when they move along the upper semi-circumference at a constant angular angle, the impulse is positive, while when they move along the lower part it has the same absolute value but the opposite sign. Therefore, the net impulse equals to zero thus no lifting force is anticipated.

This paper contributes on three topics as follows. First, it performs a study where it is assumed that the electric motors produce a variable angular velocity described by two closed-form mathematical expressions. Second, the most general case of a variable radius is also investigated. Third, the possibility of achieving a monotonically upward net impulse is discussed. Despite these novel features and the useful conclusions derived, which are applicable to several practical fields such as short-time antigravity toys, the problem of inertial propulsion is still open.

2. THEORETICAL ASPECTS

2.1. Problem definition

Let us consider an object (body B) of mass M on which two contra-rotating rigid rods (No.1 and No.2), of the same radius r, are articulated (at the points C_1 and C_2, respectively) as shown in Figure 1; the rods bring at their ends concentrated masses, of equal size m. The two aforementioned eccentric masses are driven by electric motors and rotate at constant or variable angular velocities of equal and opposite magnitude, i.e. $\omega_1(t) = -\omega_2(t) \equiv \omega$. Obviously, the mass of the electric motors is included into the body B. The initial position of the rods are denoted by ϕ_1^0 and ϕ_2^0, respectively, for which it is assumed that $\phi_1^0 = -\phi_2^0 = \phi_0$; therefore, for a later time instance, it holds that $\phi_1(t) = -\phi_2(t) = \phi(t)$. Without loss of generality, we assume that the articulation of the rigid rods is chosen at the level of the centroid G of the mass M, where also the axis origin is considered. For the sake of briefness, the problem is simplified as follows:

1. The shape of the object B appears no variation along the x- and y-axis.
2. The two masses m have the same z-coordinate.
3. The motors are fixed to the object and their shafts are parallel to the y-axis.
4. The mass of the rods and the relevant moments of inertia are neglected.

5. At the initial time instance, $t = 0$, the object is suddenly left to fall.
6. The effect of the air is neglected.
Due to the abovementioned assumptions, the components of the centrifugal forces along the x-axis are perfectly cancelled, and any possible motion of the object will be in the z-direction only. In other words, no rotation of the object B will occur, thus its vertical position can be written in the simple form:

$$z(t) = z(\omega, \phi_0). \tag{1}$$

The determination of the abovementioned function $z(t)$, and particularly the conditions for which the altitude could be increased or remain constant, is the aim of this work. It was found that the optimum condition to achieve a lift is to leave the object fall when its rods are at the horizontal position with the tendency of the rods to rotate in the upward direction. Moreover, we also investigate the possibility of using a time-varying angular velocity or/and a time-varying radius.

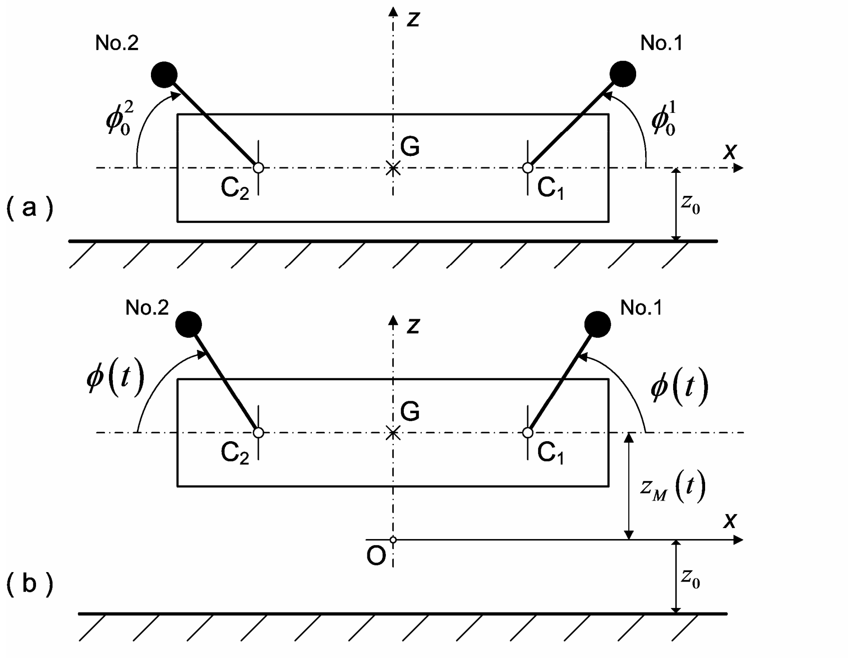

Figure 1: The setup of the object on which two contra-rotating masses (No.1 and No.2) are attached: (a) the initial position and (b) the arbitrary position of the object

2.2. Equations of motion

Due to the assumption of *massless* rods, in this paper the 'Centre of Mass Theorem' will be applied. In the general case, in order to describe rotating motion the Lagrange equation had to be used, as was done in [4,5] (these papers also used advanced mathematical analysis).

Let $\mathbf{f}_0 = f_0 \hat{\mathbf{e}}_z$ stand for the reaction force, which the ground imposes to the object, with $\hat{\mathbf{e}}_z$ denoting the unit vector of the z-axis. According to the Newton's Second Law, it holds

$$(M + 2m)\ddot{\mathbf{r}}_c = \mathbf{f}_0 + (M + 2m)\mathbf{g}. \tag{2}$$

where $(M + 2m)$ is the total mass, $\mathbf{g} = -g\hat{\mathbf{e}}_z$ is the acceleration of gravity vector, and $\ddot{\mathbf{r}}_c$ is the acceleration of the center of mass (CM) of the system of which the ordinate is given as

$$z_c = \frac{2mz_m + Mz_M}{\left(2m+M\right)} . \tag{3}$$

In Eq(3), z_M , z_m and z_c are the ordinates of the centroid of mass *M*, the masses *m* and the overall centroid, respectively. The relationship between z_M and z_m (mass No.1) is:

$$z_m = z_M + r\sin\phi . \tag{4}$$

Eliminating the ordinate z_m between Eq(3) and Eq(4), one receives:

$$z_c = z_M + \frac{2m}{\left(2m+M\right)} r\sin\phi . \tag{5}$$

In the general case of a variable radius $r = r\left(t\right)$, and taking the second temporal derivative, the vertical acceleration is given by

$$\left(2m+M\right)\ddot{z}_c = \left(2m+M\right)\ddot{z}_M + 2m\left(r\sin\phi\right)^{\cdot\cdot} . \tag{6}$$

Since the term $\left(2m+M\right)\ddot{z}_c$ represents the sum of the external forces, which consist of only the gravitational ones, $-\left(2m+M\right)g$, Eq(6) becomes

$$\left(2m+M\right)\ddot{z}_M + 2m\left(r\sin\phi\right)^{\cdot\cdot} + \left(2m+M\right)g = 0 . \tag{7}$$

Considering that the instantaneous angular velocity is defined as

$$\omega = \frac{d\phi\left(t\right)}{dt} = \dot{\phi}\left(t\right) . \tag{9}$$

the equation of motion of the free object B, an ordinary differential equation (ODE), is written in one of the following equivalent forms:

$$\ddot{z}_M = \underbrace{-\frac{2m}{\left(2m+M\right)}\left(r\sin\phi\right)^{\cdot\cdot}}_{f(t)} - g = f\left(t\right) - g . \tag{10a}$$

Provided the involved derivatives are continuous (smooth variation), the first time integration of Eq(10a) leads to:

$$\dot{z}_M\left(t\right) = \left(v_0 - gt\right) - \frac{2m}{\left(2m+M\right)}\left[\dot{r}\left(t\right)\sin\phi\left(t\right) + r\left(t\right)\dot{\phi}\left(t\right)\cos\phi\left(t\right) - \left(\dot{r}_0\sin\phi_0 + r_0\omega_0\cos\phi_0\right)\right]. \tag{10b}$$

while the second integration leads to:

$$z_M(t)=\left(z_0+v_0t-\frac{1}{2}gt^2\right)-\frac{2m}{(2m+M)}\left\{\left[r(t)\sin\phi(t)-r_0\sin\phi_0\right]-\left(\dot{r}_0\sin\phi_0+r_0\omega_0\cos\phi_0\right)t\right\}. \quad (10c)$$

It is remarkable that the altitude $z_M(t)$ includes the gravitational term $\left(z_0+v_0t-\frac{1}{2}gt^2\right)$ responsible for the free fall, a small compound term $-\frac{2m}{(2m+M)}\left[r(t)\sin\phi(t)-r_0\sin\phi_0\right]$, as well as another linear term $\left(\dot{r}_0\sin\phi_0+r_0\omega_0\cos\phi_0\right)t$. When the object starts from the rest condition ($z_0=0, v_0=0$), and the latter coefficient of t is positive, the altitude initially increases until the square term $-1/2\,gt^2$ dominates.

2.3. Particular case of constant radius

2.3.1. General

In the case of a constant radius, $r(t)=r_0$, the equations of motion obtain one of the following forms:

$$\ddot{z}_M=-\mu\left(\sin\phi\right)^{\cdot\cdot}-g, \quad (11a)$$

$$\ddot{z}_M=-\mu\left(\dot{\phi}\cos\phi\right)^{\cdot}-g, \quad (11b)$$

$$\ddot{z}_M=\mu\left[\omega^2\sin\phi-\dot{\omega}\cos\phi\right]-g. \quad (11c)$$

where ω^2 and $\dot{\omega}$ terms are related to the centrifugal and the tangential components, respectively, and

$$\mu=\frac{2mr}{(2m+M)}. \quad (12)$$

Then, the general solution becomes:

$$\begin{aligned} z_M(t)&=\left(z_0+v_0t-1/2\,gt^2\right)+\left(\mu\omega_0\cos\phi_0\right)t-\mu\left(\sin\phi-\sin\phi_0\right),\\ \dot{z}_M(t)&=\left(v_0-gt\right)-\mu\left(\omega\cos\phi-\omega_0\cos\phi_0\right). \end{aligned} \quad (13)$$

2.3.2. Analytical solution fulfilling the motion equation

In the particular case that we demand the angular velocity, $\omega(t)$, to fulfill the equation of motion (Eq(11a)), the general solution of this ordinary differential equation is given in the form

$$\phi(t)=\sin^{-1}\left[-\left(\frac{g}{2\mu}\right)t^2-c_1t+c_2\right],\quad -\frac{\pi}{2}<\phi(t)<\frac{\pi}{2} \quad (14)$$

Assuming initial conditions $\phi(0)=\phi_0$ and $\dot{\phi}(0)=\omega_0$, it implies that $c_2=\sin\phi_0$ and $c_1=-\omega_0\cos\phi_0$ ($-\pi/2<\phi_0<\pi/2$), respectively, thus Eq(14) becomes

$$\phi(t)=\sin^{-1}\left[-\left(\frac{g}{2\mu}\right)t^2+\left(\omega_0\cos\phi_0\right)t+\sin\phi_0\right],\ -\frac{\pi}{2}<\phi(t)<\frac{\pi}{2} \tag{15}$$

Moreover, in terms of the time *t*, the angular velocity is found as

$$\omega(t)=\frac{\left[-\left(\frac{g}{\mu}\right)t+\left(\omega_0\cos\phi_0\right)\right]}{\sqrt{1-\left[-\left(\frac{g}{2\mu}\right)t^2+\left(\omega_0\cos\phi_0\right)t+\sin\phi_0\right]^2}},\ -\frac{\pi}{2}<\phi(t)<\frac{\pi}{2} \tag{16}$$

2.3.3. Exponential angular velocity

Alternatively, we choose that

$$\phi(t)=\phi_0+\frac{\omega_0}{\lambda}\left(e^{\lambda t}-1\right), \tag{17}$$

whence

$$\omega(t)=\dot{\phi}(t)=\omega_0 e^{\lambda t}=\omega_0+\lambda\left(\phi-\phi_0\right), \tag{18}$$

where $\lambda=\Delta\omega/\Delta\phi$ is a constant, while ω_0 and ϕ_0 are the initial angular velocity and the initial angular (polar) position of the rods, respectively. Obviously, the relationship between time and polar angle is

$$t=\frac{1}{\lambda}\ln\left[\left(\phi-\phi_0\right)\frac{\lambda}{\omega_0}+1\right]. \tag{19}$$

Also, the temporal derivative of the angular velocity becomes:

$$\dot{\omega}=\lambda\omega_0 e^{\lambda t}=\lambda\omega. \tag{20}$$

3. NET IMPULSE

3.1. Definition of Impulse

Using the forcing function $f(t)$ of Eq(10a), it is recalled that the quantity

$$I(t)=\int_0^t f(\tau)\,d\tau \tag{21}$$

is called *impulse* and represents the change in momentum of the object during the time interval $[0,t]$. Based on this definition, apart from the gravitational term $(v_0 - gt)$, the additionally produced (propulsive) velocity will be given as

$$\left[\dot{z}_M(t)\right]_{propulsive} = \int_0^t I(\tau)d\tau = -\frac{2m}{(2m+M)}\int_0^t (r\sin\phi)^{\cdot\cdot}\,d\tau = -\frac{2m}{(2m+M)}\left[(r\sin\phi)^{\cdot}\right]_0^t. \tag{22}$$

3.2. Theorems and Lemmas

Based on the above definitions and findings, it is now possible to introduce some elementary ‘theorems’ and ‘lemmas’ to assist the researcher saving his/her efforts from ‘useless’ further studies. In the lack of available space, the text has been compressed as much as possible.

Lemma-1: In case of constant angular velocity and radius, the impulse of the centrifugal forces is zero.

Lemma-2: Under the conditions of Lemma-1, despite the fact that the impulse is zero, it is possible to obtain a significantly high altitude. In fact, the latter is due to the initial conditions and leads to a term of the first degree in time *t*, i.e. $(\dot{r}_0\sin\phi_0 + r_0\omega_0\cos\phi_0)t$ in Eq(10c). Unfortunately, after a certain time instance the second degree gravitational term $-1/2\,gt^2$ dominates and the object returns to the ground.

Theorem-1: When the rotating mass follows a permanent orbit and obtains the same velocity at the same position, after a period of time *T*, the net impulse becomes zero ($I_{net} = 0$).

In fact, for an arbitrary time instance *t*, the corresponding impulse is

$$I(t) = -\frac{2m}{(2m+M)}\int_0^t (r\sin\phi)^{\cdot\cdot}\,d\tau = -\frac{2m}{(2m+M)}\int_0^t (r\sin\phi)^{\cdot\cdot}\,\frac{d\phi}{\omega(\tau)}. \tag{23}$$

while for a complete circumference ($\phi_0 \le \phi \le 2\pi + \phi_0$), it holds that

$$I_{net} = -\frac{2m}{(2m+M)}\int_{t_0}^{t_0+T} (r\sin\phi)^{\cdot\cdot}\,d\tau = -\frac{2m}{(2m+M)}\left[(r\sin\phi)^{\cdot}\right]_{t_0}^{t_0+T}. \tag{24}$$

According to the above assumptions it holds: $r(t_0+T) = r(t_0)$ and $\sin\phi(t_0+T) = \sin\phi(t_0) = \sin\phi_0$, $\left[r(t)\sin\phi(t)\right]^{\cdot} = (r\sin\phi)^{\cdot}_{t=0}$, and therefore $I_{net} = 0$.

Lemma-3: The variation of the radius *r* does not lead to a net impulse.

Theorem-2: In case of a constant radius *r*, the change of velocity of the object depends on the change of the angular velocity. In fact, the latter becomes:

$$\dot{z}_M(t) = \underbrace{(v_0 - gt)}_{free\ fall\ term} - \mu\int_0^t (\dot{\phi}\cos\phi)^{\cdot}\,d\tau = \underbrace{(v_0 - gt)}_{free\ fall\ term} - \mu\left[\omega(\tau)\cos\phi(\tau)\right]_0^t. \tag{25}$$

Therefore, the variation of the object’s velocity between a certain position ϕ_{ref} along the circumference drawn by the rigid rods and the same position after a time interval Δt will be:

$$\Delta\dot{z}_M\left(\phi_{ref}\to\phi(t)\right)=-g\Delta t-\mu\cos\phi_{ref}\cdot\left[\omega(t)-\omega(0)\right], \tag{26a}$$

while for an entire period of time will be given as

$$\Delta\dot{z}_M\left(\phi_{ref}\to\phi_{ref}+2k\pi\right)=-gT-\mu\cos\phi_{ref}\cdot\Delta\omega_k\,,k=1,2,\dots,$$
$$\text{with }\Delta\omega_k=\omega\left(t_{ref}+kT\right)-\omega\left(t_{ref}\right) \tag{26b}$$

Lemma-4: In case of a constant radius *r*, the net impulse for a complete round of the rigid rods (measured from the initial position ϕ_0) is $I_{net}=-\mu\cos\phi_0\cdot\Delta\omega$.

Lemma-5: In case of a constant radius *r* and a repeated angular velocity (i.e. $\omega=\omega(\phi)$), it is concluded that at the end of every subsequent round, the object will obtain the velocity it would have when it would be freely left to fall.

Lemma-6: In case of a constant radius *r*, the object velocity may increase when the angular velocity changes.

4. NUMERICAL RESULTS

We consider an object of mass *M* = 5kg, on which two rotating masses (*m* = 1kg each) are attached at a distance *r*=0.1m. In all cases, the initial angular velocity is taken equal to $\omega_0=314.16\,\text{s}^{-1}=3000\,\text{rpm}$. Typical graphs of the object's velocity $\dot{z}_M(t)$ and the corresponding $\omega(t)$ are illustrated in Figure 2 and Figure 3, respectively. Finally, the vertical displacement of the object for the first four rounds of the rigid rods is shown in Figure 4.

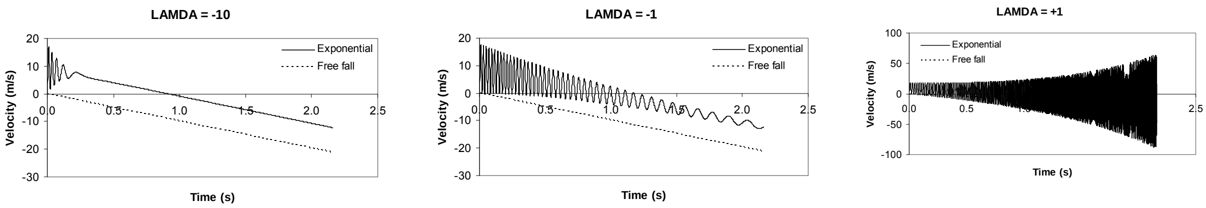

Figure 2: Object velocity, $\dot{z}_M(t)$, for three different exponents λ= -1, -10, and +1 [Eq(17)-Eq(20)]

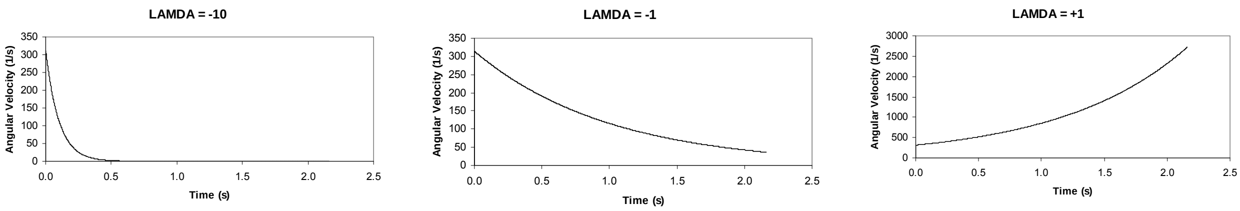

Figure 3: Angular velocity versus time, $\omega(t)$, for the three different exponents λ of Figure 2.

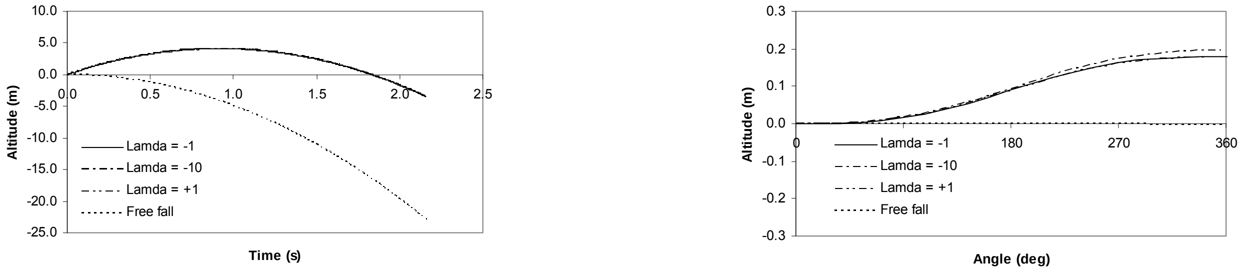

Figure 4: Vertical displacement $z_M(t)$ ***versus time for the first four rounds of the rigid rods (left) and the first one (right)***

5. CONCLUSIONS

It was shown that using two contra-rotating masses (driven by electric motors) which have reached a sufficiently *high* angular velocity, a temporal lift of the object to which they are attached may be achieved. The latter appears even if the net impulse equals to zero. Every candidate orbit of the rotating masses, which is characterized by a repeated time history of the angular velocity (the same value at the same position), leads to a zero net impulse; therefore, the object can not remain into the air for a long time. Furthermore, despite the fact that a closed-form variable angular velocity fulfilling the equation of motion was found, unfortunately it became singular in the neighborhood of the vertical positions of the rigid arms; therefore, using this formula the object can not remain still into the air. In contrast, a variable angular velocity of exponential type can achieve a net impulse for only the half of the swept circle, thus it can not solve the problem; however it is capable of controlling the velocity of the object so as it becomes adequately smooth. Finally, it was shown that a hypothetical net impulse leads to a standard quadratic term in time, which further leads to a permanent upward antigravity motion. However, the mechanism for producing such a net impulse remains still an open problem.

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Поступила в редакцию 26.03.2010
После доработки 15.04.2010